AQA A-level Mathematics (7357)

Practice Paper 1: Pure Mathematics

Time allowed: 2 hours
Total marks: 100
Calculator: Allowed

Instructions

- Answer **all** questions.
- You must show sufficient working to justify your answers.
- Unless otherwise stated, give exact values.
- You may use standard results from the A-level Mathematics formulae booklet.

This is an original practice paper written in the style of AQA A-level Mathematics.

1. (7 points) (a) Simplify fully:
$$\frac{15x^{7/2}y^{-4}}{5x^{-1/2}y^3}.$$
Give your answer in the form kx^my^n, where k is a constant.

   (b) Hence, simplify:
$$\left(\frac{15x^{7/2}y^{-4}}{5x^{-1/2}y^3}\right)^{-2}.$$
Give your answer with positive indices.

2. (10 points) (a) Simplify
$$\frac{3x^2+7x-6}{x^2-5x-6}.$$

   (b) Hence solve
$$\frac{3x^2+7x-6}{x^2-5x-6}=2.$$
Give exact solutions.

3. (10 points) The circle C has equation $x^2+y^2-6x+4y-3=0$.
   (a) Find the centre and radius of C.

   (b) Find the equation of the tangent to C at the point $(7,-2)$.

4. (9 points) A geometric sequence has first term a and common ratio r. The third term is 12 and the sixth term is 96.
   (a) Find the values of a and r.

   (b) Find the sum of the first 6 terms of the sequence.

5. (9 points) Prove the identity
$$\frac{1-\sin x}{\cos x}\equiv\frac{\cos x}{1+\sin x}.$$
(You may start from either side.)

6. (11 points) A curve has equation $y=(x^2+1)e^{-x}$.
   (a) Find $\dfrac{dy}{dx}$.

   (b) Find the equation of the tangent to the curve at $x=0$.

   (c) Show that the curve has a stationary point at $x=1$ and determine its nature.

7. (10 points) The region R is bounded by the curve $y=x^2+1$ and the line $y=2x+1$.
   (a) Find the x-coordinates of the points of intersection.

   (b) Find the exact area of region R.

8. (9 points) The curve is defined implicitly by

$$x^2 + xy + y^2 = 7.$$

   (a) Find $\dfrac{dy}{dx}$ in terms of x and y.

   (b) Find the gradient of the tangent at the point $(1, 2)$.

9. (10 points) (a) Find the first four terms in ascending powers of x of

$$(1 - 4x)^{\frac{1}{2}}.$$

   (b) Use your expansion to estimate $(0.96)^{\frac{1}{2}}$. Give your answer correct to 4 decimal places.

10. (15 points) Solve the differential equation

$$\frac{dy}{dx} = \frac{2x}{y}, \qquad y(0) = 3.$$

   (a) Solve the differential equation.

   (b) Find the value of y when $x = 2$.

   (c) Hence, find the range of values of x for which the solution is defined.