AQA A-level Mathematics (7357)

Practice Paper 1: Pure Mathematics

Time allowed: 2 hours
Total marks: 100
Calculator: Allowed

Instructions

- Answer **all** questions.
- You must show sufficient working to justify your answers.
- Unless otherwise stated, give exact values.
- You may use standard results from the A-level Mathematics formulae booklet.

This is an original practice paper written in the style of AQA A-level Mathematics.

1. (1 point) Evaluate $\sin^2 30^\circ + \cos^2 30^\circ$.
   A. 0
   B. $\dfrac{1}{2}$
   C. 1
   D. 2

2. (2 points) The minimum value of $f(x) = x^2 - 4x + 1$ is
   A. -5
   B. -3
   C. 1
   D. 3

3. (3 points) Simplify fully:
$$\frac{3\sqrt{12} + 2\sqrt{27}}{\sqrt{3}}.$$

4. (4 points) Solve exactly:
$$x^2 - 7x + 10 = 0.$$

5. (4 points) Solve for x:
$$\log_2(x - 1) + \log_2(x - 3) = 3,$$
   stating any restrictions on x.

6. (4 points) Expand $(1 - 2x)^5$ up to and including the term in x^2.

7. (5 points) Given $y = x^2 \ln x$ for $x > 0$,
   (a) Find $\dfrac{dy}{dx}$.
   (b) Hence find the gradient of the tangent at $x = e$.

8. (5 points) Evaluate:
$$\int_0^2 (3x^2 - 4x + 1)\, dx.$$

9. (5 points) Prove the identity
$$\frac{1 - \cos 2x}{\sin 2x} = \tan x.$$

10. (5 points) The line L has equation $3x - 4y + 7 = 0$.
    (a) Find the gradient of L.
    (b) Find the equation of the line perpendicular to L passing through $(2, -1)$.

11. (5 points) A geometric sequence has first term a and common ratio r. Given that the second term is 12 and the fifth term is 96, find a and r.

12. (6 points) Solve the equation
$$2\sin x = \sqrt{3}$$
    for $0 \le x < 2\pi$.

13. (6 points) The curve has parametric equations $x = t^2 + 1$, $y = 2t^3 - 3t$.

    (a) Find $\dfrac{dy}{dx}$ in terms of t.

    (b) Find the equation of the tangent when $t = 1$.

14. (6 points) (a) Express

$$\frac{5x + 1}{(x - 1)(x + 2)}$$

    in partial fractions.

    (b) Hence evaluate

$$\int \frac{5x + 1}{(x - 1)(x + 2)}\, dx.$$

15. (6 points) Find the exact area of the region enclosed by $y = 4x - x^2$ and the x-axis.

16. (7 points) Solve the differential equation

$$\frac{dy}{dx} = 3x^2 y, \qquad y(0) = 2.$$

    (a) Solve the differential equation.

    (b) Find y when $x = 1$.

17. (7 points) Let $f(x) = \dfrac{2x - 3}{x + 1}$, $x \neq -1$.

    (a) Find $f^{-1}(x)$, stating its domain.

    (b) Solve $f(x) = x$.

18. (5 points) Use two iterations of the Newton–Raphson method to find an approximation to a root of

$$x^3 - 2x - 5 = 0$$

    starting with $x_0 = 2$. Give your answer to 3 decimal places.

19. (8 points) The integral

$$I = \int_0^2 \frac{1}{x + 2}\, dx$$

    is to be estimated using the trapezium rule with step length $h = 0.5$.

x	0	0.5	1.0	1.5	2.0
$\dfrac{1}{x+2}$	0.5000	0.4000	0.3333	0.2857	0.2500

    (a) Use the trapezium rule to estimate I.

    (b) Find the exact value of I, and hence calculate the percentage error in your estimate.

20. (6 points) Prove by contradiction that $\sqrt{2}$ is irrational.