AQA A-level Mathematics (7357)

Practice Paper 2: Pure Mathematics and Mechanics

Time allowed: 2 hours
Total marks: 100
Calculator: Allowed

Instructions

- Answer **all** questions.
- You must show sufficient working to justify your answers.
- Unless otherwise stated, give exact values.
- You may use standard results from the A-level Mathematics formulae booklet.

This is an original practice paper written in the style of AQA A-level Mathematics.

Section A: Pure Mathematics

1. (1 point) Differentiate e^{3x}.

   A. e^{3x}

   B. $3e^{3x}$

   C. e^{x^3}

   D. $3xe^{3x}$

2. (5 points) Solve the inequality
$$|2x - 5| \leq 3.$$

3. (6 points) Evaluate
$$\int_1^2 \left(\frac{2}{x} + 3x^2\right) dx.$$

4. (6 points) Prove the identity
$$\sin(2x)\cot x = 2\cos^2 x.$$

5. (6 points) Vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$.

   (a) Find $\mathbf{a} \cdot \mathbf{b}$.

   (b) Hence find the angle between $\mathbf{a}$ and $\mathbf{b}$.

6. (4 points) Solve for x:
$$5^{2x} = 25^{x-1}.$$

7. (5 points) Find
$$\int \frac{6x}{x^2 + 4}\, dx.$$

8. (6 points) The curve has equation $y = x^3 - 6x^2 + 9x$.

   (a) Find the stationary points.

   (b) Determine the nature of each stationary point.

9. (5 points) A geometric series has second term 12 and third term 4.

   (a) Find the values of a and r.

   (b) Find the sum to infinity.

10. (4 points) Using the binomial expansion, show that for small x,
$$(1 + x)^{-2} \approx 1 - 2x + 3x^2.$$

11. (6 points) The function is
$$f(x) = \ln(x^2 + 1).$$

    (a) Find $f'(x)$ and $f''(x)$.

    (b) Hence find the coordinates of the point of inflection.

Section B: Mechanics

12. (1 point) A particle has velocity $v = 6t$ m s^{-1}. Its acceleration is

    A. 6 m s^{-2}

    B. $6t$ m s^{-2}

    C. $3t^2$ m s^{-2}

    D. 0 m s^{-2}

13. (5 points) A particle starts from rest and accelerates uniformly at 2 m s^{-2} for 6 s. Find its speed and the distance travelled.

14. (6 points) A projectile is launched with speed 20 m s^{-1} at an angle of 30° above the horizontal. Take $g = 9.8$ m s^{-2}.

    (a) Find the time of flight (returning to the same horizontal level).

    (b) Find the horizontal range.

15. (7 points) A particle of mass 3 kg is in equilibrium on a rough plane inclined at 25° to the horizontal. The coefficient of friction between the particle and the plane is μ.

    (a) Show that $\mu \geq \tan 25^\circ$ is required for equilibrium.

    (b) If $\mu = 0.6$, find the frictional force acting on the particle.

16. (10 points) Two particles A and B are connected by a light inextensible string passing over a smooth pulley. A has mass 4 kg and hangs freely. B has mass 6 kg and lies on a smooth horizontal table. The system is released from rest.

    (a) Find the acceleration of the system and the tension in the string.

    (b) Find the speed of the particles after 3 seconds.

    (c) Find the distance moved by B in the first 3 seconds.

17. (5 points) A uniform beam of length 4 m and weight 200 N rests horizontally on two supports at its ends. A load of 120 N is placed 1 m from the left end. Find the reactions at the two supports.

18. (7 points) A car of mass 900 kg is travelling at 15 m s^{-1}. The engine provides a constant driving force of 1800 N and the resistive force is 600 N.

    (a) Find the acceleration of the car.

    (b) Find the time taken to increase its speed from 15 to 25 m s^{-1}.

    (c) Find the distance travelled by the car while its speed increases from 15 m s^{-1} to 25 m s^{-1}.

19. (5 points) A particle moves in a straight line with constant acceleration a. Its initial velocity is u, its velocity after time t seconds is v, and it travels a distance s in that time.

    Using the equations

    $$v = u + at \quad \text{and} \quad s = \frac{1}{2}(u + v)t,$$

    derive the equation

    $$v^2 = u^2 + 2as.$$