AQA A-level Mathematics (7357)

Practice Paper 3: Pure Mathematics & Statistics

Time allowed: 2 hours
Total marks: 100
Calculator: Allowed

Instructions

- Answer **all** questions in **Section A** and **Section B**.
- You must show sufficient working to justify your answers.
- You may use the A-level Mathematics formulae booklet.

This is an original practice paper written in the style of AQA A-level Mathematics.

Section A: Pure Mathematics (60 marks)

1. (8 points) (a) Solve
$$|2x - 5| \leq 3.$$
Give your answer as an interval.

(b) Solve
$$|2x - 5| \leq 3x.$$
Give your answer as an interval.

2. (10 points) Let $f(x) = \dfrac{x-2}{x+1}$, $x \neq -1$.

(a) Find $f^{-1}(x)$.

(b) Solve $f(f(x)) = \dfrac{1}{4}$.

3. (10 points) The curve is given by $x^2y + y^2 = 8$.

(a) Find $\dfrac{dy}{dx}$ in terms of x and y.

(b) Find the equation of the tangent at the point $(2, 2)$.

4. (10 points) Evaluate
$$\int_0^{\pi/2} x \sin x \, dx.$$

5. (12 points) Solve, for $0 \leq x < 2\pi$,
$$2\cos x - \sqrt{3}\sin x = 1.$$
Give answers in exact form.

6. (10 points) The sequence is defined by $u_1 = 9$ and
$$u_{n+1} = 0.6u_n + 4.$$

(a) Show that $u_n < 10$ for all $n \geq 1$.

(b) Find the limit of u_n as $n \to \infty$.

Section B: Statistics (40 marks)

1. (12 points) A machine produces bolts. The probability that a bolt is defective is 0.08. A random sample of 80 bolts is taken. Let X be the number of defective bolts in the sample.

   (a) State the distribution of X.

   (b) Use a suitable approximation to find $P(X \leq 3)$.

   (c) Use your approximation to find $P(4 \leq X \leq 8)$.

   (d) Hence find $P(X \geq 9)$.

2. (10 points) A population has known standard deviation $\sigma = 12$. A random sample of size 36 has mean $\bar{x} = 103$. Test at the 5% level whether the population mean is greater than 100.

3. (8 points) A biased coin has probability 0.6 of landing heads. The coin is tossed 10 times. Let X be the number of heads obtained.

   (a) State the distribution of X.

   (b) Find $P(X = 7)$.

   (c) Find $P(X \leq 2)$.

   (d) Find $P(3 \leq X \leq 6)$.

4. (10 points) A survey is carried out on people who subscribe to streaming services. The events A, B and C are defined as follows:

   - A: the person subscribes to Netflix,
   - B: the person subscribes to Disney+,
   - C: the person subscribes to Amazon Prime Video.

   The following probabilities are known:

   $$P(A) = 0.55, \quad P(B) = 0.48, \quad P(C) = 0.42,$$

   $$P(A \cap B) = 0.25, \quad P(A \cap C) = 0.22, \quad P(B \cap C) = 0.18,$$

   $$P(A \cap B \cap C) = 0.10.$$

   (a) Draw a Venn diagram and label all regions with the correct probabilities.

   (b) Find $P(A \cup B \cup C)$.

   (c) Find $P(\text{exactly two of } A, B, C)$.

   (d) Find $P(A \mid B)$.