AQA A-level Mathematics (7357)

Practice Paper 3: Pure Mathematics and Statistics

Time allowed: 2 hours
Total marks: 100
Calculator: Allowed

Instructions

- Answer **all** questions.
- You must show sufficient working to justify your answers.
- Unless otherwise stated, give exact values.
- You may use standard results from the A-level Mathematics formulae booklet.

This is an original practice paper written in the style of AQA A-level Mathematics.

Section A: Pure Mathematics

1. (1 point) Evaluate $\displaystyle\int \frac{2x}{x^2+1}\,dx$.

   A. $\ln(x^2+1)+C$

   B. $\frac{2}{x^2+1}+C$

   C. $\arctan(x)+C$

   D. $(x^2+1)^2+C$

2. (5 points) Write $3x^2+6x-7$ in the form $3(x+a)^2+b$.

3. (5 points) Show that the curve with equation

$$y = \ln x - \frac{2}{x}$$

   has no stationary point for $x > 0$.

4. (4 points) Evaluate:

$$\int_0^{\pi/2} \sin x\,dx.$$

5. (6 points) The points $A(2,1,0)$ and $B(6,3,4)$.

   (a) Find the vector $\overrightarrow{AB}$.

   (b) Find the unit vector in the direction of AB.

6. (4 points) Solve exactly:

$$|x-3| = 2x-1.$$

7. (6 points) Solve for x:

$$\ln(3x-2)+\ln(x-1)=2.$$

8. (4 points) Find the coefficient of x^3 in the expansion of $(2-x)^6$.

9. (4 points) A sequence is defined by $u_1 = 5$ and $u_{n+1} = 0.8u_n + 3$. Find u_2 and u_3.

10. (5 points) The curve is $y = x^3 - 3x$. Find the equation of the tangent at the point where $x = 2$.

11. (4 points) An arithmetic sequence has first term a and common difference d.

    The 5th term is 18 and the 12th term is 39.

    (a) Find the values of a and d.

    (b) Find the sum of the first 20 terms of the sequence.

12. (12 points) Evaluate the integral

$$\int x^2 e^{2x}\,dx.$$

Section B: Statistics

13. (1 point) A fair die is rolled twice. What is the probability that the sum of the scores is a prime number?

    A. $\frac{5}{18}$

    B. $\frac{7}{18}$

    C. $\frac{1}{3}$

    D. $\frac{2}{3}$

14. (4 points) A discrete random variable X has probability function:

$$P(X = 0) = 0.2, \quad P(X = 1) = 0.5, \quad P(X = 2) = 0.3.$$

    Find $E(X)$.

15. (4 points) Events A and B satisfy $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$. Find $P(A \mid B)$.

16. (6 points) Let $X \sim B(12, 0.4)$.

    (a) Find $P(X = 5)$.

    (b) Find $P(X \geq 10)$.

17. (5 points) Let $Y \sim N(50, 6^2)$. Find $P(44 < Y < 56)$.

18. (5 points) Let $X \sim B(80, 0.3)$. Use a normal approximation with continuity correction to estimate $P(X \leq 20)$.

19. (4 points) A manufacturer claims that 30% of items produced are defective.

    In a random sample of 20 items, 10 are found to be defective.

    Test, at the 5% significance level, whether there is evidence that the proportion of defective items is greater than claimed.

20. (4 points) The table shows values of x and y.

x	1	2	3	4	5	6
y	2	4	5	7	8	9

    Using your calculator, find the product moment correlation coefficient and interpret this value.

21. (7 points) In a group of 80 students:

    - 35 study Mathematics (M),
    - 40 study Physics (P),
    - 30 study Chemistry (C),
    - 15 study both Mathematics and Physics,
    - 12 study both Mathematics and Chemistry,

- 10 study both Physics and Chemistry,
- 5 study all three subjects.

(a) Draw a Venn diagram to represent this information.

(b) Find the probability that a randomly chosen student studies exactly one of the three subjects.