AQA A-level Mathematics (7357)

Practice Paper 3: Pure Mathematics + Statistics

Time allowed: 2 hours
Total marks: 100
Calculator: Allowed

Instructions

- Answer **all** questions.
- You must show sufficient working to justify your answers.
- Unless otherwise stated, give exact values.
- You may use standard results from the A-level Mathematics formulae booklet.

This is an original practice paper written in the style of AQA A-level Mathematics.

Section A: Pure Mathematics

1. (2 points) Evaluate:
$$\frac{8!}{6!\,2!}.$$

2. (8 points) A sequence is defined by
$$u_1 = 5, \qquad u_{n+1} = 0.8u_n + 3.$$
   (a) Find u_2, u_3, u_4 and u_5.
   (b) Show that the sequence has a limiting value L and find L.
   (c) Determine the smallest value of n for which $u_n > 14$.

3. (8 points) Sketch the region satisfying
$$y \geq x^2 - 4, \qquad y \leq 2x + 2, \qquad x \geq 0.$$
   Clearly show all intersections and boundary curves.

4. (10 points) A circle has centre O and radius 8 cm.
   (a) A chord AB subtends an angle of $\frac{2\pi}{3}$ radians at the centre. Find the exact area of the minor segment cut off by the chord.
   (b) A sector of the same circle has area 48π cm^2. Find the angle subtended at the centre, in radians.

5. (6 points) Prove directly that the sum of two odd integers is even.

6. (10 points) (a) Express
$$\frac{3x + 5}{(x-1)^2(x+2)}$$
   in partial fractions.
   (b) Hence evaluate
$$\int \frac{3x + 5}{(x-1)^2(x+2)}\,dx.$$

7. (6 points) Evaluate:
$$\int \frac{4x}{x^2 + 9}\,dx.$$

8. (10 points) The curve is defined implicitly by
$$x^2 + y^2 = 6x + 2y.$$
   (a) Find $\dfrac{dy}{dx}$.
   (b) Find the equation of the tangent to the curve at the point $(2, 2)$.

Section B: Statistics

9. (4 points) (a) A researcher stands at the entrance of a library and asks every 5th person who enters to complete a questionnaire. State the sampling method used and give one advantage of this method.

   (b) Another researcher posts a survey link on their social media account and uses only the responses they receive. State the sampling method used and give one disadvantage of this method.

10. (8 points) The table shows the number of books read by a group of students in a month.

Number of books, x	$0 \le x < 2$	$2 \le x < 4$	$4 \le x < 6$	$6 \le x < 8$	$8 \le x < 10$
Frequency, f	6	10	8	4	2

    Estimate the mean, the variance and the standard deviation.

11. (8 points) A box contains 3 red balls and 2 blue balls. Two balls are drawn at random without replacement.

    (a) Draw a tree diagram.

    (b) Find the probability that exactly one ball is red.

    (c) Find the probability that the two balls are the same colour.

    (d) Find the probability that the second ball is red, given that the first ball is blue.

12. (8 points) A manufacturer claims that 90% of items are non-defective. A random sample of 20 items is tested and 15 are non-defective. Carry out a hypothesis test at the 5% significance level to assess the claim. (Use a one-tailed test and define your hypotheses clearly.)

13. (12 points) The lifetime T (in hours) of a component is modelled by

$$T \sim N(500,\ 40^2).$$

    (a) Find $P(T < 450)$.

    (b) Find $P(480 < T < 540)$.

    (c) Find $P(T > 560)$.

    (d) Find $P(T < 520)$.

    (e) Find $P(460 < T)$.

    (f) Find $P(470 < T < 530)$.