AQA A-level Mathematics (7357)

Practice Paper 2: Pure Mathematics & Mechanics

Time allowed: 2 hours
Total marks: 100
Calculator: Allowed

Instructions

- Answer **all** questions in **Section A** and **Section B**.
- You must show sufficient working to justify your answers.
- You may use the A-level Mathematics formulae booklet.
- Take $g = 9.8$ m s^{-2} where required.

This is an original practice paper written in the style of AQA A-level Mathematics.

Section A: Pure Mathematics (60 marks)

1. (8 points) Solve the inequality

$$2x + 5 \geq -x^2 + 7x + 6.$$

Give your answer as a set of values of x.

2. (10 points) The function f is defined by $f(x) = \ln(3x + 1)$ for $x > -\frac{1}{3}$.

   (a) Differentiate $f(x)$ with respect to x.

   (b) Find the equation of the tangent to $y = f(x)$ at $x = 1$.

   (c) Find the x-coordinate of the point on the curve where the gradient is $\dfrac{3}{10}$.

3. (12 points) The curve has equation $y = \dfrac{2x}{x^2 + 1}$.

   (a) Show that $\dfrac{dy}{dx} = \dfrac{2(1 - x^2)}{(x^2 + 1)^2}$.

   (b) Hence find the coordinates of the stationary points.

   (c) Sketch the curve, showing clearly the stationary points and the x- and y-intercepts.

4. (10 points) Evaluate

$$\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sin x}\, dx.$$

Give your answer in exact form.

5. (10 points) A geometric progression has first term a and common ratio r, where $0 < r < 1$. The sum to infinity is 36 and the second term is 9.

   (a) Find the values of a and r.

   (b) Find the sum of the first 30 terms of the progression, giving your answer to 3 decimal places.

6. (10 points) A curve is given by

$$y = x \cos x \quad \text{for } 0 \leq x \leq \pi.$$

   (a) Find $\displaystyle\int_0^{\pi} x \cos x\, dx$.

   (b) Hence find the exact area between the curve and the x-axis on $[0, \pi]$.

Section B: Mechanics (40 marks)

1. (10 points) A particle moves in a straight line so that its velocity at time t seconds is

$$v = 12 - 3t \quad (\text{m s}^{-1}), \qquad 0 \leq t \leq 6.$$

   (a) Find the acceleration of the particle.

   (b) Find the displacement of the particle in the first 6 seconds.

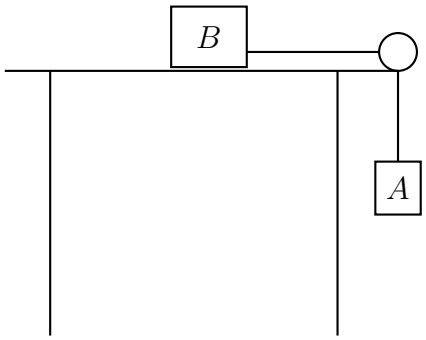
   (c) Find the total distance travelled in the first 6 seconds.

2. (10 points) A particle of mass 2 kg rests on a rough horizontal plane. A horizontal force of magnitude 18 N acts on the particle. The coefficient of friction between the particle and the plane is $\mu = 0.4$.

   (a) Determine whether the particle moves.

   (b) If the particle moves, find its acceleration.

3. (10 points) Two particles, A and B, of masses 3 kg and 2 kg respectively are connected by a light inextensible string passing over a smooth pulley. Particle A hangs freely. Particle B rests on a smooth horizontal table.



   (a) Find the acceleration of the system and the tension in the string.

   (b) State two modelling assumptions made in this question.

4. (10 points) A particle is projected with speed $20\,\text{ms}^{-1}$ at an angle of 35° above the horizontal from a point on level ground. Ignoring air resistance:

   (a) Find the time of flight.

   (b) Find the horizontal range.

   (c) Find the greatest height reached.