AQA A-level Mathematics (7357)

Practice Paper 1: Pure Mathematics

Time allowed: 2 hours
Total marks: 100
Calculator: Allowed

Instructions

- Answer **all** questions.
- You must show sufficient working to justify your answers.
- Unless otherwise stated, give exact values.
- You may use standard results from the A-level Mathematics formulae booklet.

This is an original practice paper written in the style of AQA A-level Mathematics.

1. (1 point) Simplify:
$$\frac{a^7 b^{-2}}{a^3 b^4}.$$

2. (1 point) Differentiate with respect to x:
$$y = 5\sqrt{x}.$$

3. (4 points) A circle has radius 6 cm. A sector has angle $\frac{5\pi}{6}$ radians at the centre. Find the exact length of the arc of the sector.

4. (6 points) The quadratic function is
$$f(x) = 2x^2 - 8x + 3.$$
   (a) Write $f(x)$ in the form $2(x-a)^2 + b$.
   (b) Hence state the coordinates of the turning point of the graph of $y = f(x)$.
   (c) State the minimum value of $f(x)$.

5. (7 points) (a) Expand
$$(1+3x)^{\frac{1}{2}}$$
   in ascending powers of x up to and including the term in x^3.
   (b) Use your expansion in part (a) to estimate the value of $\sqrt{1.06}$.
   (c) State the range of values of x for which the binomial expansion is valid.

6. (8 points) Let $f(x) = 2x - 5$ and $g(x) = \sqrt{x+1}$.
   (a) Find $(g \circ f)(x)$ and state its domain.
   (b) Find $(f \circ g)(x)$ and state its domain.
   (c) Find $f^{-1}(x)$.

7. (6 points) Solve, for $0 \leq x < 2\pi$,
$$2\sin x - \sqrt{3} = 0.$$

8. (6 points) Solve the equation
$$\log_3(x-1) + \log_3(x-4) = 2.$$

9. (5 points) Evaluate:
$$\sum_{r=1}^{10} (3r - 2).$$

10. (4 points) Prove by contradiction that $\sqrt{5}$ is irrational.

11. (5 points) Evaluate:
$$\int_2^5 (3x^2 - 4x + 1)\, dx.$$

12. (10 points) The curve C has equation
$$x^2 + xy + y^2 = 7.$$

(a) Find $\dfrac{dy}{dx}$ in terms of x and y.

(b) Find the gradient of the tangent to C at the point $(2, 1)$.

(c) Find the equation of the normal to C at the point $(2, 1)$.

13. (10 points) Evaluate:
$$\int \frac{4x + 1}{x^2 + x - 2}\, dx.$$

14. (5 points) The curve has parametric equations
$$x = 4 + t, \qquad y = 2t^2 - 1, \qquad -3 \leq t \leq 2.$$
Find a Cartesian equation of the curve in the form $y = f(x)$ and state the domain of f.

15. (6 points) Consider the function
$$y = x^3 - 6x^2 + 9x + 1.$$

(a) Find the coordinates of the stationary points.

(b) Determine the nature of each stationary point.

16. (6 points) The integral
$$I = \int_1^3 \ln x\, dx$$
is to be estimated using the trapezium rule with step length $h = 0.5$.

x	1.0	1.5	2.0	2.5	3.0
$\ln x$	0.0000	0.4055	0.6931	0.9163	1.0986

(a) Use the trapezium rule to estimate I.

(b) The exact value is $3 \ln 3 - 2$. Find the percentage error (to 2 d.p.).

17. (10 points) Evaluate:
$$\int \frac{x^2}{(x^3 + 1)\sqrt{x^3 + 1}}\, dx.$$