AQA A-level Mathematics (7357)

Practice Paper 3: Pure Mathematics + Statistics

Time allowed: 2 hours
Total marks: 100
Calculator: Allowed

Instructions

- Answer **all** questions.
- You must show sufficient working to justify your answers.
- Unless otherwise stated, give exact values.
- You may use standard results from the A-level Mathematics formulae booklet.

This is an original practice paper written in the style of AQA A-level Mathematics.

Section A: Pure Mathematics

1. (2 points) Evaluate:
$$\binom{9}{2}.$$

2. (8 points) A sequence is defined by
$$u_1 = 2, \qquad u_{n+1} = 0.6u_n + 4.$$
   (a) Find u_2 and u_3.
   (b) Show that
   $$u_{n+1} - 10 = 0.6(u_n - 10).$$
   (c) Hence show that the sequence converges, and find its limiting value.

3. (8 points) Sketch the region satisfying
$$y \le x^2, \qquad y \ge x + 2, \qquad x \le 3.$$
   Clearly show all intersections and boundary curves.

4. (8 points) A circle has centre O and radius 10 cm. A chord AB subtends an angle of $\frac{\pi}{2}$ radians at the centre.
   (a) Find the exact area of the minor segment cut off by the chord.
   (b) Find the exact perimeter of the minor segment.

5. (4 points) Prove directly that the product of two consecutive integers is even.

6. (8 points) (a) Express
$$\frac{5x - 1}{(x+1)(x-2)}$$
   in partial fractions.
   (b) Hence evaluate
   $$\int_0^1 \frac{5x - 1}{(x+1)(x-2)}\,dx.$$

7. (6 points) Evaluate:
$$\int \frac{6x}{x^2 + 4}\,dx.$$

8. (8 points) The curve is defined implicitly by
$$x^2 + 2y^2 = 8x - 4y.$$
   (a) Find $\dfrac{dy}{dx}$.
   (b) Find the equation of the tangent at the point $\left(2,\, -1 + \sqrt{7}\right)$.

Section B: Statistics

9. (4 points) A college has a list of 600 students in alphabetical order. A student chooses a random starting point in the first 12 names, then selects every 12th student after that until 50 students have been chosen.
   (a) State the sampling method.
   (b) Explain why this sampling method may not give a representative sample.

10. (8 points) The numbers of goals scored by 12 players in a season are:

$$0, 1, 2, 0, 3, 4, 1, 2, 5, 2, 1, 3.$$

   (a) Find the mean and the variance.
   (b) Two more players join the team. Each of these players scored 6 goals. Find the new mean and the new variance.

11. (6 points) A box contains 4 green balls and 3 yellow balls. Two balls are drawn at random without replacement.
   (a) Draw a tree diagram to represent this situation.
   (b) Find the probability that the two balls are the same colour.
   (c) Given that at least one of the balls drawn is green, find the probability that both balls are green.

12. (10 points) A company claims that 30% of its customers purchase an extended warranty. A random sample of 25 customers is taken and 11 purchased an extended warranty. Carry out a hypothesis test at the 5% significance level to assess the claim. (Use a two-tailed test and define your hypotheses clearly.)

13. (10 points) The mass M (in grams) of a certain type of chocolate bar is modelled by

$$M \sim N(52,\ 1.8^2).$$

   (a) Find $P(M < 50)$.
   (b) Find $P(51.5 < M < 54.5)$.
   (c) Find the value of m such that $P(M < m) = 0.90$.
   (d) Find the value of k such that $P(M > k) = 0.15$.
   (e) Find $P(|M - 52| < 1.5)$.

14. (10 points) The heights (in cm) of a group of students are:

$$171,\ 148,\ 163,\ 156,\ 160,\ 205,\ 155,\ 169,\ 172,\ 162,\ 150,\ 174,\ 158,\ 165,\ 166.$$

   (a) Find the minimum, Q_1, Q_2, Q_3, the maximum and the interquartile range.
   (b) Determine whether there are any outliers using the $1.5 \times IQR$ rule.
   (c) Draw a box plot to represent this data on a suitable scale.