AQA A-level Mathematics (7357)

Practice Paper 2: Pure Mathematics + Mechanics

Time allowed: 2 hours
Total marks: 100
Calculator: Allowed

Instructions

- Answer **all** questions.
- You must show sufficient working to justify your answers.
- Unless otherwise stated, give exact values.
- You may use standard results from the A-level Mathematics formulae booklet.

This is an original practice paper written in the style of AQA A-level Mathematics.

Section A: Pure Mathematics

1. (1 point) Solve the equation
$$x^2 - 7x + 10 = 0.$$
   A. $x = -2$ or $x = -5$
   B. $x = 2$ or $x = 5$
   C. $x = 1$ or $x = 10$
   D. $x = \dfrac{7 \pm \sqrt{39}}{2}$

2. (1 point) Find the coefficient of x^3 in the binomial expansion of
$$(2 - 3x)^6.$$
   A. 720
   B. -810
   C. -4320
   D. 4320

3. (1 point) The curve has equation $y = 4x - x^2$. The area under the curve between $x = 0$ and $x = 4$ is
   A. 0
   B. $\dfrac{32}{3}$
   C. 16
   D. $\dfrac{64}{3}$

4. (6 points) (a) (3 points) Sketch the graph of
$$y = x^2 - 6x + 5.$$
   Your sketch must clearly show the coordinates of all intercepts with the axes and the turning point.

   (b) (3 points) Hence (or otherwise) sketch the graph of
$$y = \left|x^2 - 6x + 5\right| + 2x - 5.$$
   Your sketch must clearly show where the modulus causes a change in the form of the graph, and include any intercepts.

5. (2 points) Show that for any integer n, the expression
$$n^2 + 3n$$
   is even.

6. (7 points) The curve C is defined implicitly by
$$x^2 + y^2 + xy = 7.$$

(a) (3 points) Find $\dfrac{dy}{dx}$ in terms of x and y.

(b) (4 points) Find the coordinates of the stationary points of C.

7. (8 points) A function f is such that, for $x > 0$,

$$f'(x) = (x^2 - 2x + 2)e^x + 2x \ln x,$$

and $f(1) = 0$. Find $f(x)$.

8. (10 points) The horizontal displacement x (in metres) of a point on a rotating mechanism is modelled by

$$x = 7\sin\theta - 4\cos\theta,$$

where θ is measured in radians.

(a) (5 points) Show that x can be written in the form

$$x = R\sin(\theta - \beta),$$

where $R > 0$ and $0 < \beta < \dfrac{\pi}{2}$, stating the exact values of R and β.

(b) (2 points) Hence find the maximum value of x for $0 \leq \theta < 2\pi$.

(c) (3 points) Hence solve, for $0 \leq \theta < 2\pi$,

$$7\sin\theta - 4\cos\theta = 3.$$

Give your answers in exact form.

9. (14 points) A curve C lies in the region $x > 0$.

At a general point $P(x, y)$ on C, the gradient of the tangent is directly proportional to y and inversely proportional to x.

The constant of proportionality is 2.

(a) (3 points) Show that y satisfies the differential equation

$$\frac{dy}{dx} = \frac{2y}{x}.$$

(b) (4 points) Solve the differential equation.

(c) (3 points) The curve passes through the point $(1, 3)$. Find the equation of C.

(d) (4 points) Find the coordinates of the point on C where the gradient of the tangent is equal to 12.

Section B: Mechanics

10. (1 point) A resultant force of 12 N acts on a particle of mass 3 kg. The magnitude of its acceleration is

    A. 0.25 m s^{-2}

    B. 4 m s^{-2}

    C. 9 m s^{-2}

    D. 36 m s^{-2}

11. (1 point) A force of 18 N acts at a perpendicular distance 0.25 m from a pivot. The moment of the force about the pivot is

    A. 0.72 N m

    B. 4.0 N m

    C. 4.5 N m

    D. 72 N m

12. (5 points) A particle moves along a straight line. Its velocity-time graph consists of two straight-line sections:

    - from $t = 0$ to $t = 4$, the velocity increases uniformly from 0 to 12 m s^{-1};
    - from $t = 4$ to $t = 10$, the velocity decreases uniformly from 12 m s^{-1} to -6 m s^{-1}.

    (a) (2 points) Find the time at which the particle changes direction.

    (b) (3 points) Find (i) the displacement, and (ii) the total distance travelled, in the first 10 seconds.

13. (8 points) A particle of mass 6 kg rests on a rough horizontal surface. It is pulled by a force of magnitude P acting at an angle of 25° above the horizontal. The coefficient of friction between the particle and the surface is $\mu = 0.30$. Take $g = 9.8$ m s^{-2}.

    (a) (4 points) Show that if the particle is on the point of moving, then

    $$P = \frac{\mu m g}{\cos 25^\circ + \mu \sin 25^\circ}.$$

    (b) (4 points) When $P = 25$ N, find the acceleration of the particle.

14. (6 points) A particle moves along a straight line with acceleration

    $$a = 6t - 2 \quad (\text{m s}^{-2}),$$

    where t is measured in seconds. At $t = 0$ the velocity is -1 m s^{-1} and the displacement is 0.

    (a) (3 points) Find the velocity v as a function of t.

    (b) (3 points) Find the displacement s when the particle first comes instantaneously to rest.

15. (9 points) A car is travelling along a straight road. At $t = 0$ its speed is 10 m s^{-1}. It then accelerates uniformly at 2 m s^{-2} until its speed reaches 26 m s^{-1}. It continues at this constant speed for 12 seconds, and then decelerates uniformly at 3 m s^{-2} until it comes to rest.

    (a) (2 points) Find the time taken for the car to accelerate from 10 m s^{-1} to 26 m s^{-1}.

    (b) (2 points) Find the distance travelled while the car moves at the constant speed.

    (c) (5 points) Find the total distance travelled from $t = 0$ until the car comes to rest.

16. (6 points) A ball is projected vertically upwards from a point 5 m above the ground with speed 14 m s^{-1}. Take $g = 9.8$ m s^{-2}.

    (a) (4 points) Find the time taken for the ball to hit the ground.

    (b) (2 points) Find the speed of the ball as it hits the ground.

17. (14 points) A car A of mass 1200 kg pulls a trailer B of mass 800 kg along a straight horizontal road. The engine provides a constant driving force of 2500 N. The resistive forces are constant and have magnitudes 400 N on A and 600 N on B. The car and trailer are connected by a light inextensible tow-bar, and the system moves in a straight line. Initially, the system is at rest.

    (a) (4 points) Find the acceleration of the car-trailer system.

    (b) (4 points) Find the magnitude of the force in the tow-bar.

    (c) (4 points) After 10 seconds, the driving force is reduced to 1500 N (resistive forces unchanged). Find the new acceleration immediately after the change, and state whether the speed is increasing or decreasing at that instant.

    (d) (2 points) Using your answer to part (a), find the speed of the system after the first 10 seconds, and the distance travelled in that time.