AQA A-level Mathematics (7357)

Practice Paper 3: Pure Mathematics + Statistics

Time allowed: 2 hours
Total marks: 100
Calculator: Allowed

Instructions

- Answer **all** questions.
- You must show sufficient working to justify your answers.
- Unless otherwise stated, give exact values.
- You may use standard results from the A-level Mathematics formulae booklet.

This is an original practice paper written in the style of AQA A-level Mathematics.

Section A: Pure Mathematics

1. (1 point) The circle has equation

$$x^2 + y^2 - 6x + 4y - 12 = 0.$$

Its centre is

A. $(6, -4)$

B. $(3, -2)$

C. $(3, -2)$

D. $(-3, 2)$

2. (1 point) The curve has equation $y = 2x^4 - 3x^2 + 5x$. The gradient of the tangent at $x = -1$ is

A. -9

B. 3

C. 9

D. -1

3. (1 point) Line l_1 has equation $3y - 2x = 7$. Line l_2 is perpendicular to l_1. The gradient of l_2 is

A. $\dfrac{3}{2}$

B. $-\dfrac{2}{3}$

C. $-\dfrac{3}{2}$

D. $\dfrac{2}{3}$

4. (3 points) Sketch the graph of

$$y = |2x - 3| - 2\,|x + 1| + 1.$$

Your sketch must clearly show all points where the gradient changes and any intercept(s) with the axes.

5. (3 points) Show that for small values of θ (in radians),

$$\frac{\tan\theta}{1 + \cos\theta} \approx \frac{\theta}{2},$$

and hence state the equation of the straight line that approximates the graph of $y = \dfrac{\tan\theta}{1 + \cos\theta}$ near $\theta = 0$.

6. (13 points) (a) (2 points) A function g is defined by

$$g(x) = \frac{\sqrt{2x - 1}}{(x - 2)\ln(x + 3)}.$$

State the **maximum possible domain** of g.

(b) (4 points) A function f is defined by

$$f(x) = \frac{x^2 + 1}{x^2 - 4x + 5}.$$

Use the **quotient rule** to find $\dfrac{df}{dx}$ and simplify your answer.

(c) (4 points) Hence find the stationary points of f and determine the nature (maximum/minimum) of each stationary point.

(d) (3 points) Write down the set of values of x for which the graph of $y = f(x)$ is **convex**.

7. (5 points) For $a > 0$ and $a \neq 1$,

$$\log_a(2a - 3) - \log_a(a - 1) = \log_a 4 - 1.$$

Rearrange this equation to make a the subject, giving the exact value(s) of a for which the equation is valid.

8. (9 points) (a) (4 points) Prove the identity

$$\frac{1}{1 + \sin x} \equiv \sec^2 x - \sec x \tan x.$$

(b) (5 points) Hence evaluate

$$\int_0^{\pi/2} \frac{1}{1 + \sin x}\, dx.$$

9. (7 points) A geometric sequence has first term a and common ratio r. It is known that the second term is 6 and the fifth term is 48.

(a) (3 points) Find the exact values of a and r.

(b) (2 points) Find an expression for the sum of the first n terms, S_n, and hence find S_8.

(c) (2 points) Find the least value of n such that the nth term exceeds 3000.

10. (7 points) Prove by contradiction that $\sqrt[3]{3}$ is irrational.

Section B: Statistics

11. (1 point) A discrete random variable X has the probability distribution:

x	0	1	2	3
$P(X = x)$	0.12	0.25	k	0.18

The value of k is

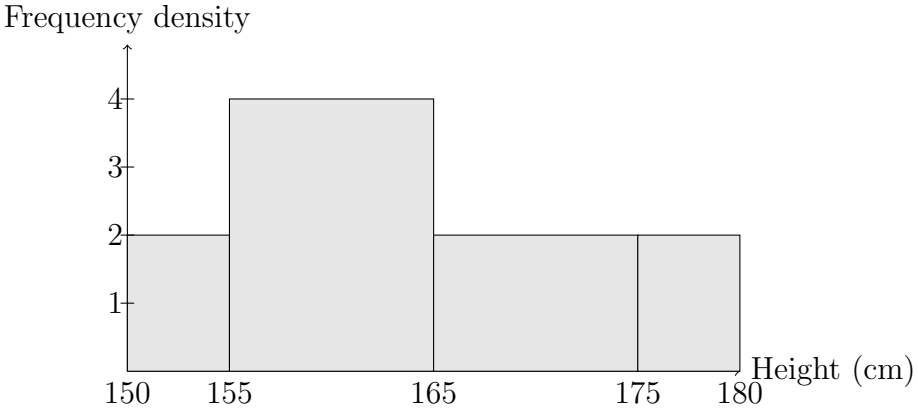
A. 0.57

B. 0.33

C. 0.45

D. 0.20

12. (1 point) A histogram shows the heights (cm) of 80 students.



The class interval that contains the median height is

A. $150 \leq h < 155$

B. $155 \leq h < 165$

C. $165 \leq h < 175$

D. $175 \leq h < 180$

13. (3 points) The table shows grouped data for the times (minutes) taken by 30 students to complete a quiz.

Time t (min)	$0 \leq t < 10$	$10 \leq t < 20$	$20 \leq t < 30$	$30 \leq t < 40$
Frequency	5	9	12	4

Estimate the standard deviation of the times.

14. (6 points) In a college of 180 students, three subjects are studied: Mathematics (M), Physics (P) and Chemistry (C).

$$|M| = 92, \quad |P| = 85, \quad |C| = 70,$$

$$|M \cap P| = 40, \quad |M \cap C| = 35, \quad |P \cap C| = 30,$$

and 25 students study none of these subjects.

(a) (3 points) Draw a Venn diagram to represent this information, showing the number in each region.

(b) (3 points) A student is known to study *exactly two* of the subjects. Find the probability that C is one of the subjects studied.

15. (7 points) In a factory, each item produced is defective with probability p, independently of other items. Each day a sample of 12 items is tested, and X is the number of defective items found.

(a) (2 points) State the distribution of X.

(b) (2 points) It is found that $P(X = 0) = 0.2824$ (to 4 d.p.). Use this to estimate p (to 3 s.f.).

(c) (1 point) Using your value of p, find $P(X \geq 2)$ (to 3 s.f.).

(d) (1 point) Find $E(X)$.

(e) (1 point) Using your value of p, find the least value of n such that for $Y \sim \text{Bin}(n, p)$,

$$P(Y \geq 1) > 0.95.$$

16. (12 points) A random variable X is normally distributed with mean μ and standard deviation σ. It is given that

$$P(X < 40) = 0.20 \quad \text{and} \quad P(X > 70) = 0.10.$$

(a) (4 points) Show that μ and σ satisfy two linear equations and hence find μ and σ (to 3 s.f.).

(b) (3 points) Find $P(45 < X < 65)$ (to 3 s.f.).

(c) (2 points) A new variable is defined by $Y = 2X - 5$. State the distribution of Y.

(d) (3 points) Find $P(Y > 120)$ (to 3 s.f.).

17. (12 points) A basketball player claims that the probability of scoring a free throw is 0.75. In each test, the player takes 20 free throws and X is the number scored.

(a) (6 points) In a recent test the player scored 11 out of 20. Test at the 5% significance level whether there is evidence that the player's success probability has *decreased*. State your hypotheses clearly, find an appropriate critical region, and give your conclusion.

(b) (6 points) After a training programme, another test is carried out (again 20 free throws) and the player scores 19 out of 20. Test at the 5% significance level whether there is evidence that the player's success probability has *increased*. State your hypotheses clearly, find an appropriate critical region, and give your conclusion.

18. (8 points) A student surveys opinions about the school canteen by asking the first 60 students who enter the dining hall at lunchtime.

(a) (1 point) State the sampling method used.

(b) (1 point) State one reason why this sample should not be used to carry out a hypothesis test about the opinions of *all* students in the school.

(c) (6 points) A different survey is carried out properly. The time (in minutes) taken to travel to school is modelled by a normal distribution with known standard deviation 12 minutes. A random sample of 36 students has mean travel time 42 minutes. Test, at the 5% significance level, whether there is evidence that the mean travel time is *less than* 45 minutes. You should state your hypotheses, calculate an appropriate test statistic, and state your conclusion.